International Journal of Engineering Research & Management Technology
ISSN: 2348-4039
Volume 9, Issue-3 May-June 2022
Email:editor@ijermt.org www.ijermt.org

VEDIC MATHEMATICS IN QUADRATIC, CUBIC AND QUARTIC EQUATIONS

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Abstract: Vedic mathematics is found to be very effective and sound for mental calculations in mathematics. Sutras and sub sutras have beautiful and striking tricks for fast and easy for mathematical calculations. In this article, we explore on importance of Vedic Mathematics with thematic analysis. Vedic Math provides more systematic, simplified, unified and faster than the conventional system. A significant and interesting invention which has led to various applications in all the disciplines is the development of Vedic Math approach.

Keywords: *Quadratic, Vedic Mathematics*

QUADRATIC EQUATIONS

The general quadratic equation in variable x with the constants a, b, and c is

$a.x^2 + b.x + c = 0, a \neq 0$

The quadratic formula is the current approach for solving quadratic equations.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Two values of x can be discovered using the formula above. The Vedic method, which is superior to the existing method, can also be used to solve the quadratic problem.

Solve:

$z + 1/z = 10/3$

By using current method,

Taking L.C.M. we get,

$3z^2 + 3 = 10z$

$\therefore 3z^2 - z - 9z + 3 = 0$

$\therefore z (3z – 1) – 3 (3z – 1) = 0$

$\therefore (z – 3) (3z – 1) = 0$

$z = 3$ or $z = 1/3$

∴ Roots of given Quadratic equation are z = 3 or z = 1/3

By using Vilokanam,

The right-hand term is 10/3, which can be represented as a two-step addition.

Reciprocals 3 + 1/3

∴ z + 1 z = 3 + 1/3

∴ z = 3 or z = 1/3

∴ Roots of given Quadratic equation are z = 3 or z = 1/3

Second Special Type under Śūnyam Sāmyasamuccaye Sūtra

To solve the quadratic equation of the type

$$\frac{kx+1}{mx+n} = \frac{px+q}{rx+s}$$

Where (k + p) = (m + r) & (l + q) = (n + s);

We can apply the Śūnyam Sāmyasamuccaye Sūtra in the following way:

Consider the oneness of the total and equating it to 0, which yields the provided quadratic's one root.

i.e. (k x + l) + (p x + q) = (m x + n) + (r x + s) = 0

Which yields one of the quadratic equation's roots, consider the oneness of the difference and assume that it equals 0; this yields another root for the given quadratic equation.

i.e. (k x + l) - (m x + n) = (p x + q) – (r x + s) = 0;

Solve the following problem:

$$\frac{5z+2}{2z+4} = \frac{3z+5}{6z+3}$$

By using Śūnyam Sāmyasamuccaye Sūtra,

Consider the oneness of the total and make the sum = 0, which gives the supplied quadratic one root.

Here, Sum = (5z + 2) + (3z + 5) = (2z + 4) + (6z + 3) = 8z + 7

8z + 7 = 0

which implies z = -7/8

Consider the oneness of two sides' differences. Equivalently, divide the difference by 0 to get another root for the given quadratic equation.

i.e. (5z + 2) – (2z + 4) = (6z + 3) – (3z + 5) = 0

Therefore z = 2/3

∴Roots of given Quadratic equation are

z = 2/3

or

z = -7/8

$$\frac{11y+2}{13y+5}=\frac{9y+7}{7y+4}$$

By using, Śūnyam Sāmyasamuccaye Sūtra,

Consider the sum's oneness; equating sum = 0 yields one root of the supplied quadratic.

i.e. (11y + 2) + (9y + 7) = (13y + 5) + (7y + 4) = 0

20y + 9 = 0

$$\therefore y=-\frac{9}{20}$$

Consider the oneness of the difference between the two sides; equating that difference to 0 yields a new root for the quadratic equation.

i.e. (13y + 5) – (11y + 2) = (7y + 4) – (9y + 7) = 0

2y + 3 = 0

$$\therefore y=-\frac{3}{2}$$

∴ Roots of given Quadratic equation are

$$\therefore y=-\frac{9}{20}$$

By using Vedic Sūtra:

$$\frac{3}{w+3}+\frac{4}{w+4}=\frac{5}{w+5}+\frac{2}{w+2}$$

Here, both the conditions are satisfied.

$$\frac{3}{1}+\frac{4}{1}=\frac{5}{1}+\frac{2}{1}\ and\ \frac{3}{3}+\frac{4}{4}=\frac{5}{5}+\frac{2}{2}$$

∴ According to the Sunyamanyat Sūtra,

One root of given equation is w = 0.

Also, (w + 3) + (w + 4) = (w + 5) + (w + 2) = 2w + 7

∴ as per Śūnyam Sāmyasamuccaye Sūtra,

That addition = 2w + 7 = 0

∴the another root is w = −7/2

∴ The solution w = 0 or w = − 7/2

Factorization of a one-variable quadratic expression

If we want to factorize a quadratic expression with a prefix of x and the highest degree 2 as unity, we must use the existing system. in order to solve quadratics in one variable $Ax^2 + Bx + C$, divide the term B in parts such that addition of that two numbers = the coefficient of variable x and the independent quantity equals the multiplication value.

Factorize: $z^2 + z - 12$

Here, co-proficient of z^2 is solidarity. Gap the co-effective of z (for example +1) in two sections with the end goal that expansion of that is +1 and the augmentation of two sections is - 12.

Here, (+ 4 – 3) = + 1

& (+ 4) (- 3) = - 12

$z^2 + z - 12$

$= z^2 + 4z - 3z - 12$

$= z(z + 4) - 3(z + 4)$

$= (z - 3)(z + 4)$

We can figure the result of (z - 3) and (z + 4) orally, which checks to $z^2 + z - 12$. Elements of 12 are 1, 2, 3, 4, 6, and 12.

In the event that we select + 4 and - 3 whose aggregate is + 1 and item is - 12.

We can confirm it by in an upward direction and transversely Sūtra

$z + 4$

$$\frac{z-3}{z^2+z-12}$$

Thus, Factors of quadratic expression $z^2 + z - 12 = (z + 4)(z - 3)$

Factorization of Quadratics expression with four variables

Settling quadratic equations containing multiple factors is a serious testing task particularly for the equations in homogeneous structure.

Let t, u, v and w be four factors.

Vedic Ādyamadyenāntya mantyena and Lopanasthāpanābhyām Sub-Sūtra can be used to tackle mind boggling issues in a somewhat simpler way.

Procedure of factorization of long homogeneous quadratic expression with four variables x, y, z & w:

To factorize long homogeneous quadratic articulation with four factors x, y, z and w, multiple times by eradicating two letters all at once

Either eradicate x, y by putting x = y = 0, then, at that point we will get the articulation in other two leftover factors z and w.

Eradicate y, w by putting y = w = 0 hold just in x, z or take out x, w by putting

x = w = 0 and hold just in y, z.

First eradicate x, y and hold just in z, w; take out y, z by hold just in x, w; or by disposing of x and z to hold it in y and w.

First eradicate x, w; then, at that fact take out z, w, and subsequently place of x, z.

First eradicate y, w; then, at that point take out z, w and afterward dispose of y, z.

In the wake of eradicating two letters all at once, the quadratic with staying two letters are acquired. By factorizing and blending it alongside filling the holes the last factors in each of the 4 letters can be found.

First eliminate x, w by putting x = w = 0 and retain only in y, z;

$[T] = 7y^2 + 10yz + 3z^2$

$= (y + z) (7y + 3z)$

By eliminating z & w (i.e. z = w = 0)

$[T] = 2x^2 - 9xy + 7y^2$

$= (x - y) (2x - 7y)$

$= (y - x) (7y - 2x)$

If x = z = 0, then

$[T] = 7y^2 - 2yw - 5w^2$

$= (y - w) (7y + 5w)$

With these three sets of factors,

Thus, $T = (y + z) (7y + 3z)$

$= (y - w) (7y + 5w)$

Filling in the gaps,

The solution is $T = (y + z - x - w) (7y + 3z - 2x + 5w)$

Factorization of Cubic Expressions:

Factorize: $x^3 + 2x^2 - 23x - 60$ Let $E = x^3 + 2x^2 - 23x - 60$

Sum of coefficient of all the term = 1 + 2 - 23 - 60 = - 80

Factors of 80 = 1, 2, 4, 5, 8, 10, 16, 20, 40, 80

And Factors for 60 = 1, 60, 2, 30, 3, 20, 4, 15, 5, 12, 6, 10

We want to find the factors such that product of any three is 60

& total of them must be 2.

∴ Possible factors are 3, 4,-5

Also co-efficient of x^2 + 12 - 20 - 15 = - 23

$\therefore E = x^3 + 2x^2 - 23x - 60 = (x + 3) (x + 4) (x - 5)$

FACTORIZATION OF CUBIC EQUATIONS

We can likewise settle cubic equation by utilizing Parāvartya Sūtra, Lopanasthā panābhyām Sub-Sūtra, and Pūrana pūrnabhyām Sūtra which implies by finish just as deficiency from second to fourth powers.

Solve: $z^3 + 5z^2 - z - 5 = 0$

$\therefore z^3 + 5z^2 = z + 5$

But $(z + 2)^3 = z^3 + 6z^2 + 12z + 8$

By Pūraapūrabhyām,

∴ Substituting the value of z3 + 6z2 from above, we have:

$(z + 2)^3 = z^2 + z + 5 + 12z + 8$

$= z^2 + 13z + 13$

$= (z + 2)(z + 2 + 9) - 9(z + 2)3$

By substituting $z + 2 = y$, $y^3 = y(y + 9)$

∴ $y^3 = y^2 + 9y - 9$

∴ $y^3 - y^2 - 9y + 9 = 0$

Sum of co-efficient of all the terms = 1 - 1 - 9 + 9 = 0

∴ (y-1) is one factor of this cubic equation.

By artificial division we can find other two factors (y + 3) and (y - 3).

Thus, y =1 and y = ± 3 but z + 2 = y

∴ z + 2 = 1 & z + 2 = ± 3

Thus, final solution is z = -1, z = 1 & z = -5

FACTORIZATION OF BIQUADRATIC EQUATIONS

Solve: $z^4 + 3z^3 - 21z^2 - 83z - 60 = 0$

Here, The sum of co-eff. of odd power of z = the sum of co-eff. of even power of z

∴ 3 - 83 = 1 - 21- 60

∴ - 80 = - 80

∴ (z + 1) is one factor of given equation.

By using Synthetic division,

$z^4 + 3z^3 - 21z^2 - 83z - 60$

$= (z + 1)(z^3 + 2z^2 - 23z - 60)$

Now we factorize, $(z^3 + 2z^2 - 23z - 60)$

± 1, ± 60, ± 2, ± 30, ± 3, ± 20, ± 4, ±15, ± 5, ±12, ± 6, ±10 are factors of 60.

Yet, we need three numbers with the end goal that their complete ought to be 2.

We select the gathering 3, 4,- 5 [since, 3 + 4 - 5 = 2] Also testing for the co-proficient of z.

Here, a .b + b .c + c .a = 12 - 20 -15 = - 23 = co-efficient of z.

∴ Possible factors of - 60 = (3) (4) (-5)

$z^3 + 2z^2 - 23z - 60 = (z + 3)(z + 4)(z - 5)$

∴ Given expression = $z^4 + 3z^3 - 21z^2 - 83z - 60 = (z + 1)(z + 3)(z + 4)(z + 5) = 0$

Thus, the final solution is z = -1, -3, -4 and -5.

APPLICATIONS OF DETERMINANTS

Use of Determinant in solving Simultaneous Linear Equations

Solve the following simultaneous linear equations:

"5x + 3y + z = 17

3x + 2y + 2z = 10

7x + 8y + z = 26"

Consider the fourth order determinant

$x.D$	$y.D$	$z.D$	$-D$		
–1	3	2	–1	10	–3
–10	5	3	2	17	–16
19	7	8	1	26	35

For obtaining D, we use the cross-product approach to assess the 3*3 matrix for persisting 3 columnar cells, which covers the fourth column. We've got

i.e. -1 × 1 + -10 × 2 + 19 × 1 = -1 - 20 + 19 = - 2

Cover the first column and use the cross-product approach to assess the 3 by 3 determinant of the following three columns.

i.e. 2 × 35 + 3 × (-16) + 8 × (-3) = 70 – 48 - 24 = -2

To evaluate -y, D cover the second column and use the cross-product technique to assess the 3*3 matrix for persisting 3 columnar cells.

i.e. 3 × 35 + 5 × (-16) + 7 × (-3) = 105 - 80 – 21 = 4

Cover the third column and use the cross-product approach to assess the 3*3 matrix for persisting 3 columnar cells.

i.e. (-1) × 26 + (-10) × 17 + 19 × 10 = - 26 - 170 + 190 = - 6

Thus D = -2, x. D = - 2, -y. D = 4 and z. D = - 6.

∴ The final solution is x = 1, y = 2 & z = 3

References:

1. V. B. L. Chaurasia and V. Gill, Aleph-function and equation of integral blood pressure, ActaUniversitatisApulensis, No. 35/2013, pp. 211-214, ISSN: 1582-5329.
2. Y. Singh and H. Mandia, On Some multidimensional fractional integral operators associated with generalized hypergeometric function, International Journal of Engineering Research and Applications (IJERA), ISSN: 2248- 9622 Vol. 2, Issue 3, May-Jun 2012, pp.843-849
3. A. Faraj, T. Salim, S. Sadek and J. Smail, Generalized Mittag-Leffler Function Associated with Weyl Fractional Calculus Operators, Hindawi Publishing Corporation Journal of Mathematics, Volume 2013, Article ID 821762, 5 pages.

4. D. Kumar and J. Daiya, Generalized Fractional Differentiation of the $\bar{H}$ –Function Involving General Class of Polynomials, Int. J. Pure Appl. Sci. Technol., 16(2) (2013), pp. 42-53, ISSN 2229 – 6107
5. D. Kumar and J. Singh, New Fractional-Calculus Results Involving General Class of Multivariable Polynomials and Multivariable H-function, International Journal of Modern Mathematical Sciences, 2013, 7(1): 55- 64, ISSN:2166-286X
6. D. A. Bensona, M. M. Meerschaertb, and J. Reviellea, Fractional calculus in hydrologic modeling: A numerical perspective, Adv Water Resour. 2013 January 1; 51: 479–497. doi:10.1016/j.advwatres.2012.04.005
7. F. Ahmad, D. K. Jain and R. Jain, International Mathematical Forum, Vol. 8, 2013, no. 25, 1199 – 1204, HIKARI Ltd, www.m-hikari.com
8. J. Choi and P. Agarwal, Certain unified integrals associated with Bessel functions, Springer, Boundary Value Problems 2013, 2013:95 doi:10.1186/1687-2770-2013-95.
9. R. K. Saxena, J. Ram and D. Kumar, Sumudu and Laplace Transforms of the Aleph-Function. Caspian Journal of Applied Mathematics, Ecology and Economics, ISSN 1560-4055, V. 1, No 2: 19-28, December, 2013.
10. V. Kumar, N-fractional calculus of general class of functions and Fox's H-Function. Proc. Natl. Acad. Sci., India, Sect. A Phys. Sci. (July-Sept 2013) 83(3):271-277